Laws of Exponents and Radicals

Let a and b be real numbers and m and n be rational numbers. Then the following properties of exponents hold, provided that all of the expressions appearing in a particular equation are defined.

1. $a^m a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $(ab)^m = a^m b^m$
4. $a^0 = 1$, for $a \neq 0$
5. $\dfrac{a^m}{a^n} = a^{m-n}$, for $a \neq 0$
6. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$, for $b \neq 0$
7. $a^{-m} = \dfrac{1}{a^m}$, for $a \neq 0$
8. $a^{\frac{1}{n}} = \sqrt[n]{a}$, where n is an integer
9. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$, where m and n are integers
10. $\sqrt[m]{a} \cdot \sqrt[m]{b} = \sqrt[m]{ab}$
11. $\sqrt[m]{\dfrac{a}{b}} = \dfrac{\sqrt[m]{a}}{\sqrt[m]{b}}$, for $b \neq 0$
12. $\sqrt[n]{a^n} = |a|$, if n is even
13. $\sqrt[n]{a^n} = a$, if n is odd
14. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Note: formulas 8 – 14 hold only for values of a, b, m, and n for which all of the radicals appearing are defined.